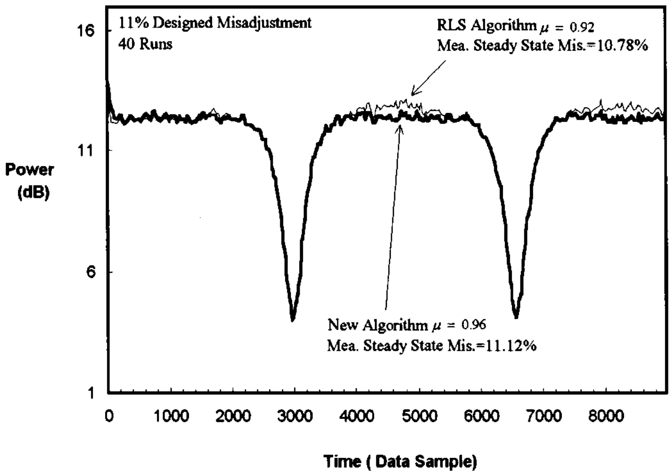

Fig. 7. Output power trajectories of the two algorithms in the scenario of Fig. 6.

curves demonstrates the much better tracking performance of the new algorithm, despite its higher computational requirements.

In general, with an implicit assumption of a linearly changing environment, the new algorithm has a better performance when the environment changes in a nonabrupt or fairly smooth manner so that the change is approximately linear in nature. This is demonstrated in the results presented when the environment does not change in a strictly linear manner all the time, but the change is smooth enough that it can be taken to be linear over the effective number of data samples used to update the weights. Specifically, based on the feedback factor used in the four-element array at 10% misadjustment above, the effective number of data samples for updating the weights is of the order of 100. If the array is mounted on a rotating platform with a speed of 100°/s and the data rate is 1 Msamples/s, this corresponds to a change of about 0.1°. Clearly, in applications such as this, the environment can be taken to be approximately linear, and the new algorithm will be quite effective.

V. CONCLUSION

A fast algorithm for the tracking of nonstationary environments in power inversion array has been presented and studied. The algorithm is based on the use of iterative least squares techniques and the introduction of an additional gradient vector in the objective function so that both the optimal weight vector and its rate of change are estimated recursively. Although the algorithm is computationally more complex than conventional least squares methods, it has better performance in tracking moving sources as the rate of change of the optimal weight vector is also being estimated.

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Further Results on Reconstruction Methods for Processing Finite-Length Signals with Perfect Reconstruction Filter Banks

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***Abstract*—In a previous paper, new expressions were developed for the perfect reconstruction (PR) of the boundary regions of a finite-length signal after subband processing using uniform and paraunitary filter banks (PUFB's). The present paper points out an incorrect assertion in the previous paper regarding the sufficiency of its solution and presents the general existence (applicability) conditions for the method. Furthermore, we extend the method to encompass perfect reconstruction filter banks, i.e., not only the paraunitary case.**

***Index Terms*—Filter banks, finite signals, transforms.**

I. INTRODUCTION

In [1], a solution was presented for the perfect reconstruction (PR) of the boundary regions of a finite-length signal after subband processing using uniform paraunitary filter banks (PUFB's). The solution was based on a formulation of a linear system whose solution yields the undistorted boundary samples of the signal. It was stated in the Appendix that said solution could always be applied, regardless of signal extension and filter bank. Here, we show that such a statement is incorrect.[1] In fact, PR is only conditionally assured. We will explain in detail the conditions to be met. The FB's for which [1] does not apply are uncommon; however, we will present a counter-example. In deriving the existence conditions, we decided to extend the solution to also accommodate bi-orthogonal (PR but not PU) FIR FB's. This will provide for completeness of the solution.

This correspondence is, in fact, an extension of [1] from which it inherits most of its notation and to where we send the readers for references. As in [1], $\boldsymbol{I}_n$ and $\boldsymbol{J}_n$ are the $n \times n$ identity and reversing matrices, respectively.

II. TRANSFORM MATRIX

There is an analysis FIR filter bank [2] with M filters $f_i(n)$ of maximum length L and a synthesis filter bank with M filters $g_i(n)$ of max-

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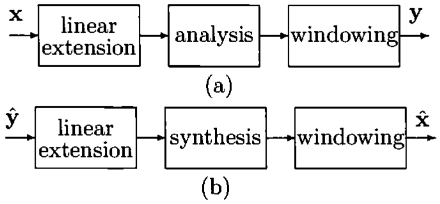

Fig. 1. Extension and windowing in the analysis and synthesis of a finite-length signal. (a) Overall analysis section. (b) Overall synthesis section.

imum length L. It is assumed that $L = KM$ for K integer and zeros can be freely appended to impulse responses to meet such assumptions. The filter coefficients are grouped into lapped transform matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$ of size $M \times L$, i.e.,

$$\boldsymbol{P} = \{p_{ij}\}, \quad p_{ij} = f_i(L-1-j), \quad \boldsymbol{Q} = \{q_{ij}\}, \quad q_{ij} = g_i(j) \tag{1}$$

$$\boldsymbol{P} = [\boldsymbol{P}_0, \boldsymbol{P}_1, \cdots, \boldsymbol{P}_{K-1}], \quad \boldsymbol{Q} = [\boldsymbol{Q}_0, \boldsymbol{Q}_1, \cdots, \boldsymbol{Q}_{K-1}]. \tag{2}$$

where $\boldsymbol{P}_i$ and $\boldsymbol{Q}_i$ are $M \times M$ matrices. The PR conditions [2], [3] establish that

$$\sum_{k=0}^{K-1-m} \boldsymbol{Q}_k^T \boldsymbol{P}_{k+m} = \sum_{k=0}^{K-1-m} \boldsymbol{Q}_{k+m}^T \boldsymbol{P}_k = \delta(m)\boldsymbol{I}_M. \tag{3}$$

Here, as in [1], we assume the model of extension and windowing described in Fig. 1. Although general at first glance, the limitations of the proposed algorithm are linked to the limitations of the framework in Fig. 1.

The input vector $\boldsymbol{x}$ is assumed to have $N_x = NM$ samples and is divided into three sections $\boldsymbol{x}^T = [\boldsymbol{x}_l^T, \boldsymbol{x}_c^T, \boldsymbol{x}_r^T]$, where $\boldsymbol{x}_l$ and $\boldsymbol{x}_r$ contain the first and last S samples of $\boldsymbol{x}$, respectively. Following the signal extension model, $\boldsymbol{x}$ is extended into $\tilde{\boldsymbol{x}}$ as

$$\tilde{\boldsymbol{x}}^T = [\boldsymbol{x}_{e,l}^T, \boldsymbol{x}^T, \boldsymbol{x}_{e,r}^T] = [(\boldsymbol{R}_l x_l)^T, \boldsymbol{x}_l^T, \boldsymbol{x}_c^T, \boldsymbol{x}_r^T, (\boldsymbol{R}_r \boldsymbol{x}_r)^T] \tag{4}$$

i.e., the extended sections are found by a linear transform of the boundary samples of $\boldsymbol{x}$ as

$$\boldsymbol{x}_{e,l} = \boldsymbol{R}_l \boldsymbol{x}_l, \quad \boldsymbol{x}_{e,r} = \boldsymbol{R}_r \boldsymbol{x}_r \tag{5}$$

and $\boldsymbol{R}_l$ and $\boldsymbol{R}_r$ are arbitrary $S \times S$ "extension" matrices. For example, $\boldsymbol{R}_l = \boldsymbol{R}_r = \boldsymbol{J}_S$ [1] yields a symmetric extension.

The transformation from the $N_x + 2S$ samples in $\tilde{\boldsymbol{x}}$ to vector $\boldsymbol{y}$ with $NM = N_x$ subband samples is achieved through the block-banded matrix $\tilde{\boldsymbol{P}}$, i.e.,

$$\tilde{\boldsymbol{P}} = \begin{pmatrix} \ddots & & & \ddots & & & & 0 \\ & \boldsymbol{P}_0 & \boldsymbol{P}_1 & \cdots & \boldsymbol{P}_{N-1} & & & \\ & & \boldsymbol{P}_0 & \boldsymbol{P}_1 & \cdots & \boldsymbol{P}_{N-1} & & \\ & & & \boldsymbol{P}_0 & \boldsymbol{P}_1 & \cdots & \boldsymbol{P}_{N-1} & \\ 0 & & & & \ddots & & & \ddots \end{pmatrix}. \tag{6}$$

Note that there are N block rows and that $S = (K-1)M/2$. Using the same notation for $\tilde{\boldsymbol{Q}}$ with respect to $\boldsymbol{Q}_i$, the analysis and synthesis systems are given by

$$\tilde{\boldsymbol{y}} = \tilde{\boldsymbol{P}}\tilde{\boldsymbol{x}} \tag{7}$$

$$\bar{\tilde{\boldsymbol{x}}} = \tilde{\boldsymbol{Q}}^T \tilde{\boldsymbol{y}} = \tilde{\boldsymbol{Q}}^T \tilde{\boldsymbol{P}} \tilde{\boldsymbol{x}} = \tilde{\boldsymbol{H}} \tilde{\boldsymbol{x}} \tag{8}$$

where $\bar{\tilde{\boldsymbol{x}}}$ is the reconstructed vector in the absence of quantization. From (3), we can easily show that the transfer matrix is

$$\tilde{\boldsymbol{H}} = \tilde{\boldsymbol{Q}}^T \tilde{\boldsymbol{P}} = \left[\begin{array}{c|c|c} \boldsymbol{H}_L & & 0 \\ \hline & \boldsymbol{I}_{N_x - 2S} & \\ \hline 0 & & \boldsymbol{H}_R \end{array}\right] \tag{9}$$

where $\boldsymbol{H}_l$ and $\boldsymbol{H}_r$ are $2S \times 2S$ matrices. Thus, distortion is just incurred to the S boundary samples in each side of $\boldsymbol{x}$ ($2S$ samples in each side of $\tilde{\boldsymbol{x}}$).

III. Recovering Distorted Samples

Let it be shown in (10) and (11) at the bottom of the page that

$$\boldsymbol{H}_l = \boldsymbol{\Psi}_l^T \boldsymbol{\Phi}_l, \quad \boldsymbol{H}_r = \boldsymbol{\Psi}_r^T \boldsymbol{\Phi}_r. \tag{12}$$

If we divide $\bar{\tilde{\boldsymbol{x}}}$ in the same manner as $\tilde{\boldsymbol{x}}$, i.e., $\bar{\tilde{\boldsymbol{x}}} = [\bar{\boldsymbol{x}}_{e,l}^T, \bar{\boldsymbol{x}}_l^T, \bar{\boldsymbol{x}}_c^T, \bar{\boldsymbol{x}}_r^T, \bar{\boldsymbol{x}}_{e,r}^T]$, then

$$\begin{aligned} \begin{bmatrix} \bar{\boldsymbol{x}}_{e,l} \\ \bar{\boldsymbol{x}}_l \end{bmatrix} &= \boldsymbol{H}_l \begin{bmatrix} \boldsymbol{x}_{e,l} \\ \boldsymbol{x}_l \end{bmatrix} \\ &= \boldsymbol{H}_l \begin{bmatrix} \boldsymbol{R}_l \boldsymbol{x}_l \\ \boldsymbol{x}_l \end{bmatrix} \\ &= \boldsymbol{H}_l \begin{bmatrix} \boldsymbol{R}_l \\ \boldsymbol{I}_S \end{bmatrix} \boldsymbol{x}_l \\ &= \boldsymbol{\Lambda}_l \boldsymbol{x}_l \end{aligned} \tag{13}$$

where

$$\boldsymbol{\Lambda}_l = \boldsymbol{H}_l \begin{bmatrix} \boldsymbol{R}_l \\ \boldsymbol{I}_S \end{bmatrix} \tag{14}$$

$$[\boldsymbol{\Phi}_l | \boldsymbol{\Phi}_r] = \left[\begin{array}{cccc|cccc} \boldsymbol{P}_0 & \boldsymbol{P}_1 & \cdots & \boldsymbol{P}_{K-2} & \boldsymbol{P}_{K-1} & & & 0 \\ & \boldsymbol{P}_0 & \boldsymbol{P}_1 & \cdots & \boldsymbol{P}_{K-2} & \boldsymbol{P}_{K-1} & & \\ & & \ddots & \ddots & & \ddots & \ddots & \\ 0 & & & \boldsymbol{P}_0 & \boldsymbol{P}_1 & \cdots & \boldsymbol{P}_{K-2} & \boldsymbol{P}_{K-1} \end{array}\right] \tag{10}$$

$$[\boldsymbol{\Psi}_l | \boldsymbol{\Psi}_r] = \left[\begin{array}{cccc|cccc} \boldsymbol{Q}_0 & \boldsymbol{Q}_1 & \cdots & \boldsymbol{Q}_{K-2} & \boldsymbol{Q}_{K-1} & & & 0 \\ & \boldsymbol{Q}_0 & \boldsymbol{Q}_1 & \cdots & \boldsymbol{Q}_{K-2} & \boldsymbol{Q}_{K-1} & & \\ & & \ddots & \ddots & & \ddots & \ddots & \\ 0 & & & \boldsymbol{Q}_0 & \boldsymbol{Q}_1 & \cdots & \boldsymbol{Q}_{K-2} & \boldsymbol{Q}_{K-1} \end{array}\right] \tag{11}$$

is a $2S \times S$ matrix. Iff $\mathbf{\Lambda}_l$ has rank S, then $\boldsymbol{x}_l$ can be recovered through the pseudo-inverse of $\mathbf{\Lambda}_l$

$$\boldsymbol{x}_l = \mathbf{\Lambda}_l^+ \begin{bmatrix} \overline{\boldsymbol{x}}_{e\,l} \\ \overline{\boldsymbol{x}}_l \end{bmatrix} = (\mathbf{\Lambda}_l^T \mathbf{\Lambda}_l)^{-1} \mathbf{\Lambda}_l^T \begin{bmatrix} \overline{\boldsymbol{x}}_{e,\,l} \\ \overline{\boldsymbol{x}}_l \end{bmatrix}. \tag{15}$$

For the other ("right") border, the identical result is trivially found to be

$$\boldsymbol{x}_r = \mathbf{\Lambda}_r^+ \begin{bmatrix} \overline{\boldsymbol{x}}_r \\ \overline{\boldsymbol{x}}_{e,\,r} \end{bmatrix} = (\mathbf{\Lambda}_r^T \mathbf{\Lambda}_r)^{-1} \mathbf{\Lambda}_r^T \begin{bmatrix} \overline{\boldsymbol{x}}_r \\ \overline{\boldsymbol{x}}_{e,\,r} \end{bmatrix} \tag{16}$$

wherein

$$\mathbf{\Lambda}_r = \boldsymbol{H}_r \begin{bmatrix} \boldsymbol{I}_S \\ \boldsymbol{R}_r \end{bmatrix} \tag{17}$$

is also assumed to have rank S. It is necessary that $\mathbf{\Phi}_l$,$\mathbf{\Phi}_r$,$\mathbf{\Psi}_l$, and $\mathbf{\Psi}_r$ have rank S but not sufficient since rank can be reduced by the matrix products. It is also possible to express in more detail the conditions, but no useful analytical solution for the filter bank could be achieved. In this case, assuming the rank checking is to be done numerically, further detailing the existence conditions would have little practical use.

Summarizing, the steps to achieve PR for given $\boldsymbol{R}_l$ and $\boldsymbol{R}_r$ are as follows.

- Construct $\boldsymbol{P}$ and $\boldsymbol{Q}$ as in (1), (2).
- Find $\mathbf{\Phi}_l$, $\mathbf{\Phi}_r$, $\mathbf{\Psi}_l$, $\mathbf{\Psi}_r$ from (10) and (11).
- Find $\boldsymbol{H}_l$ and $\boldsymbol{H}_r$ from (12).
- Find $\mathbf{\Lambda}_l$ and $\mathbf{\Lambda}_r$ from (14) and (17).
- Test rank of $\mathbf{\Lambda}_l$ and $\mathbf{\Lambda}_r$.
- If ranks are S, obtain $\mathbf{\Lambda}_l^+$, $\mathbf{\Lambda}_r^+$, and reconstruct $\boldsymbol{x}_l$ and $\boldsymbol{x}_r$.

This is the extension of [1] to non-PU (but PR) filter banks with the particular concern to test whether the pseudo inverses exist.

IV. Counter Example

In [1], it was erroneously stated that the matrices have full rank, regardless of the PU filter bank and extension matrices ($\boldsymbol{R}$). Indeed, the linear system is consistent for most cases of interest. However, counter examples can be generated. Let the polyphase transfer matrix [2] of a PU filter bank with N_c channels be $\boldsymbol{F}(z) = \boldsymbol{A} + \boldsymbol{B}z^{-1}$, where $\boldsymbol{A}$ and $\boldsymbol{B}$ are $N_c \times N_c$ matrices. Let N_c be less than $M/2$. Consider a memoryless unitary transform $\boldsymbol{D}$ and an FB given by

$$\begin{aligned} \boldsymbol{E}(z) &= \begin{bmatrix} \boldsymbol{F}(z) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{M-N_c} \end{bmatrix} \\ \boldsymbol{D} &= \begin{bmatrix} \boldsymbol{F}(z) & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{M-N_c} \end{bmatrix} \\ \begin{bmatrix} \boldsymbol{D}_1 \\ \boldsymbol{D}_2 \end{bmatrix} &= \begin{bmatrix} \boldsymbol{A}\boldsymbol{D}_1 + \boldsymbol{B}\boldsymbol{D}_1 z^{-1} \\ \boldsymbol{D}_2 \end{bmatrix} \end{aligned} \tag{18}$$

where $\boldsymbol{D}_1$ contains the top N_c rows of $\boldsymbol{D}$, whereas $\boldsymbol{D}_2$ contains the remaining rows. Since the FB is PU, then

$$\begin{aligned} \mathbf{\Phi}_l = \mathbf{\Psi}_l = \boldsymbol{P}_0 = \boldsymbol{Q}_0 &= \begin{bmatrix} \boldsymbol{A}\boldsymbol{D}_1 \\ \boldsymbol{D}_2 \end{bmatrix} \\ \mathbf{\Phi}_r = \mathbf{\Psi}_r = \boldsymbol{P}_1 = \boldsymbol{Q}_1 &= \begin{bmatrix} \boldsymbol{B}\boldsymbol{D}_1 \\ \boldsymbol{0} \end{bmatrix}. \end{aligned} \tag{19}$$

The filter bank has order 1, and thus, $K = 2$ and $S = M/2$. As $rank(\mathbf{\Phi}_r) < S$, $\boldsymbol{x}_r$ cannot be recovered using $\mathbf{\Lambda}_r$. Note that there would be no distortion if we first implemented $\boldsymbol{D}$ over the finite-length signal and then processed the output for $\boldsymbol{F}(z)$ using the method given in the previous section. However, direct implementation of $\boldsymbol{E}(z)$ fails. The reason is that different filters would require different extensions during the analysis process; therefore, the model in Fig. 1 is not applicable.

V. Conclusions

The method for boundary sample recovery was extended to encompass PR (biorthogonal) systems while noting its necessary conditions. The model in Fig. 1 and the proposed method are not applicable for several FB's, including those whose filters have different lengths and different symmetries. Examples are some two-channel biorthogonal FB's and composite systems, i.e., cascaded systems such as the counter example or as in the method given in [4]. For the second example, the proposed method can be efficiently used to implement each stage of the cascade. The method works very well for M-channel filter banks whose filters have same length. The phase of the filters and the extensions can be arbitrary, and the method has been shown to be consistent for all uniform-length FB's tested.

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Nonlinear Filtering via Generalized Edgeworth Series and Gauss–Hermite Quadrature

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***Abstract*—In this correspondence, an approximate nonlinear filter is presented for systems with continuous time dynamics and discrete time measurements. The filter is based on a combination of generalized Edgeworth series (GES) expansion of probability density functions and Gauss–Hermite quadrature (GHQ). Application to a passive tracking problem is also presented.**

***Index Terms*—Fokker–Planck–Kolmogorov equation (FPKE), Gauss–Hermite quadrature (GHQ) method, generalized Edgeworth series (GES), nonlinear filters.**

I. Introduction

The design of optimal nonlinear filters for state estimation of stochastic dynamical systems has been an area of intense research [13], [15] ever since Kalman published his celebrated work. The optimal nonlinear filter for a general nonlinear filtering problem is usually infinite dimensional if one resorts to moment evolution-based methods

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